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MODELING THE DEFORMABILITY OF A MATERIAL

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The high-temperature extension of metals to fracture is modeled for creep at constant stress or at constant strain rate. The dependence of the ultimate fracture strain on the loading factor (stress or strain rate) is studied. The nonmonotonic nature of this dependence with an internal maximum is described using the Rabotnov kinetic theory with one and two damage parameters. Available experimental data are analyzed.

Key words: *deformability, strain rate, fracture, creep, creep rupture strength, modeling, damage, superplasticity.*

High-temperatures tension tests of metals are usually performed with a specified dependence of the axial stress σ on time t or a specified dependence of the axial strain rate $\dot{\varepsilon}$ on t. At present, there is a relatively small amount of reliable experimental data on the high-temperature deformation of metals to fracture over a wide range of stresses σ or strain rates $\dot{\varepsilon}$. This is due to difficulties in strain measurements at high temperatures and difficulties that arise when the ultimate strain ε_* accumulated by the moment of fracture t_* reaches tens and hundreds percent. Because of insufficient experimental data, the equations describing the deformation of materials to fracture have not been finally formulated even for the case of uniaxial stress state.

1. Modeling the Nonmonotonic Dependence of Ultimate Creep Strain on Stress. To describe high-temperature extension to fracture, we use the concept of the mechanical equation of state proposed by Rabotnov [1] for creep processes. According to this concept, the creep strain rate $\dot{p}$ is determined by the stress σ, temperature, and a certain number of structural parameters, which vary during creep according to kinetic equations. The creep rupture strength is usually described using the structural parameter $\omega(t)$ which is a measure of damage to the material. Each state of damage is assigned a value $0 \leqslant \omega \leqslant 1$; the value $\omega = 0$ corresponds to the intact material, and value $\omega = 1$ to the fractured material.

In some creep-rupture tests of metals, the ultimate creep strain p_* at the time of fracture t_* behaves nonmonotonically in the examined range of constant tensile stresses σ_0 (see, for example, [2]). Mishchenko et al. [3], studying the creep of 15Kh1M1F steel at a temperature of 565°C analyzed the structure of the fractured samples and showed that the extrema of the dependence $p_*(\sigma_0)$ and the dependence of the number of cracks per unit surface $m(\sigma_0)$ were reached for the same value of σ_0. Lokoshchenko and Shesterikov [4] noted that in modeling the nonmonotonic dependence $p_*(\sigma_0)$, one should take into account the effect of stresses on the creep rate and the rate of damage accumulation by using various functional relations.

To describe the creep process at constant stress σ_0 to fracture and determine the strain p_*, we consider the power-law dependence of the creep rate on stress

$$\dot{p} = A\Big(\frac{\sigma_0}{1-\omega}\Big)^n \tag{1}$$

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TABLE 1

Values of $\varepsilon_*(\dot\varepsilon_0)$

$\dot\varepsilon_0$, $\sec^{-1}$	l_*/l_0	ε_*	$\varepsilon_*(\dot\varepsilon_0)$	
			Formula (10)	Formula (15)
0.0005	6.49	1.87	2.06	1.90
0.001	7.93	2.07	2.17	2.04
0.003	9.33	2.23	2.19	2,27
0,006	7,39	2,00	2,10	1,96
0,01	6,05	1,80	1.98	1.76
0.03	5.08	1.63	1.60	1.40
0.06	3.16	1.15	1.31	1.21
0.10	2.80	1.03	1.09	1.09
0.18	2.47	0.91	0.84	0.96

and three types of kinetic equation for $\dot\omega$:

$$\dot\omega = B_1 \frac{\sinh\,(\sigma_0/c)}{(1-\omega)^n}; \tag{2}$$

$$\dot\omega = B_2 \frac{\sigma_0^{n/2}\exp\,(\sigma_0/c)}{(1-\omega)^n}; \tag{3}$$

$$\dot\omega = B_3\Big(\frac{\sigma_0}{\sigma_b-\sigma_0}\Big)^k \frac{1}{(1-\omega)^n} \qquad \text{at} \quad n>k>0, \quad 0<\sigma_0<\sigma_b \tag{4}$$

(σ_b is the conditional limit of the short-term creep rupture strength at the test temperature). Integration of the relations $\dot p/\dot\omega$ according to (1)–(4) yields the following dependences of the ultimate strain p_* on the stress level σ_0:

$$p_* = \frac{A}{B_1}\frac{\sigma_0^n}{\sinh\,(\sigma_0/c)}, \qquad p_* = \frac{A}{B_2}\frac{\sigma_0^{n/2}}{\exp\,(\sigma_0/c)}, \qquad p_* = \frac{A}{B_3}\,\sigma_0^{(n-k)}(\sigma_b-\sigma_0)^k. \tag{5}$$

For small values of σ_0, all dependences $p_*(\sigma_0)$ in (5) are increasing, and for large values of σ_0, they are decreasing. Hence, for a certain intermediate value of the stress σ_{00}, the ultimate strain is maximal. In view of (5), the condition $(dp_*/d\sigma_0)\Big|_{\sigma_0=\sigma_{00}} = 0$ allows one to determine these values of σ_{00} for the kinetic equations (2)–(4), respectively:

$$\tanh\,\frac{\sigma_{00}}{c} = \frac{\sigma_{00}}{nc}, \qquad \sigma_{00} = \frac{nc}{2}, \qquad \sigma_{00} = \frac{(n-k)\sigma_b}{n}.$$

2. Modeling the Nonmonotonic Dependence $\varepsilon_*(\dot\varepsilon_0)$. Of the main requirements for the mechanical characteristics of materials, the high strength and the high deformability of a material are of great significance. Determining the conditions necessary to achieve a high level of ultimate strains (the strains characterizing the fracture of the material) is the most important for the commercial alloys used in pressure treatment of metals. The largest ultimate strains occur in the superplastic state, which is attained at a temperature and strain rate characteristic of each alloy. The mechanical behavior of materials in the superplastic state is described in detail in [5].

Results of an experimental study of the deformation to fracture of 01570 aluminum alloy at a temperature of 500°C are given in [6]. The tests were performed on flat samples cut from a sheet along the rolling direction. It is of interest to determine the strain rates leading to the maximum fracture strain at a temperature of 500°C. In those experiments, performed at a constant logarithmic strain rate $\dot\varepsilon_0$, a nonmonotonic dependence of the ultimate logarithmic strain ε_* on the value $\dot\varepsilon_0$ (with an internal maximum) was obtained. The ratio of the lengths l_* of the fractured samples to the initial length l_0 was in the range of 2.5–9.3. Table 1 gives averaged values of the parameter $\varepsilon_*(\dot\varepsilon_0)$ from 3–7 tests; in Fig. 1, they are shown by points. From the results of the tests in [6] it follows that at a temperature of 500°C and $\dot\varepsilon_0 = 0.003$ $\sec^{-1}$, 01570 alloy is deformed under superplasticity conditions.

Let us consider modeling of the results of tests in which the logarithmic strain rate $\dot\varepsilon(t)$ retains a constant value $\dot\varepsilon_0$ to fracture and the dependence of the ultimate strain ε_* on the strain rate $\dot\varepsilon_0$ is nonmonotonic with an

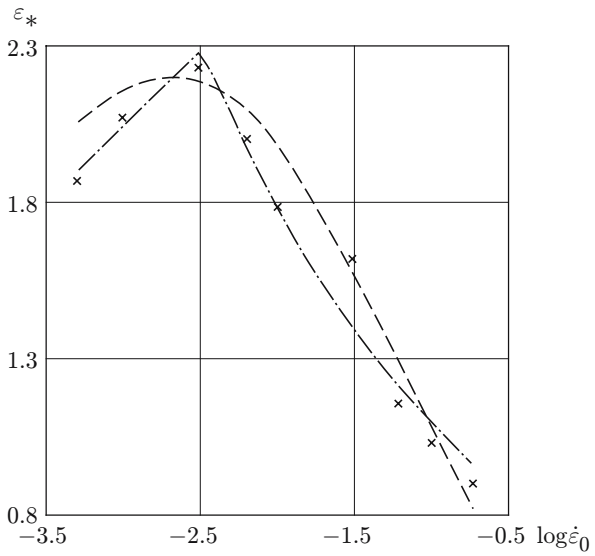

Fig. 1. Experimental (points) and calculated (curves) dependences of the ultimate strain on the logarithmic strain rate: the dashed curve refers to calculations using formula (10) for $\dot\varepsilon_{00} = 0.002$ sec^{-1}, $n = 0.15$, $\beta = 8.47$ sec^n, and $C = 0.723 \cdot 10^{-3}$ sec$^{-1/2}$) and the dot-and-dashed curve refers to calculations using formula (15) for $\dot\varepsilon_0 = 0.003$ sec^{-1}, $D = 1.32 \cdot 10^{-3}$ sec^{-1}, $n = 1.21$, and $k = 0.90$.

internal maximum. If $\omega(t)$ is a single structural parameter, the kinetic equation describing its time variation can generally be written as

$$\dot\omega = \varphi(\dot\varepsilon_0, \omega), \qquad \omega(0) = 0, \quad \omega(t_*) = 1. \tag{6}$$

We first consider the dependence of the function φ on its arguments $\dot\varepsilon_0$ and ω (6) in the form of the exponential functions

$$\dot\omega = B\dot\varepsilon_0^k (1-\omega)^{-r}. \tag{7}$$

In this case, integration of (7) yields

$$\varepsilon_* = \dot\varepsilon_0^{(1-k)}/(B(r+1)). \tag{8}$$

According to (8), for $0 < k < 1$, the strain ε_* is a monotonically increasing function of the strain rate $\dot\varepsilon_0$, and for $k > 1$, it is a monotonically decreasing function. Thus, the nonmonotonic dependence $\varepsilon_*(\dot\varepsilon_0)$ cannot be described by means of the kinetic equation (7).

We replace the exponential dependence $\dot\omega(\dot\varepsilon_0, \omega)$ in (7) by a function with three constants:

$$\dot\omega = C\sqrt{\dot\varepsilon_0}\exp(\beta\dot\varepsilon_0^n). \tag{9}$$

Then, the dependence of the ultimate strain ε_* on the rate $\dot\varepsilon_0$ becomes

$$\varepsilon_* = \sqrt{\dot\varepsilon_0}\,/(C\exp(\beta\dot\varepsilon_0^n)). \tag{10}$$

For small values of $\dot\varepsilon_0$, the dependence $\varepsilon_*(\dot\varepsilon_0)$ is increasing, and for large values of $\dot\varepsilon_0$, it is decreasing. Hence, there exists an intermediate value $\dot\varepsilon_{00}$ for which the dependence $\varepsilon_*(\dot\varepsilon_0)$ has a maximum. The condition $(d\varepsilon_*/d\dot\varepsilon_0)\Big|_{\dot\varepsilon_0=\dot\varepsilon_{00}} = 0$ implies that the quantities $\dot\varepsilon_{00}$, n, and β are linked by the relation

$$2\beta n(\dot\varepsilon_{00})^n = 1.$$

Table 1 gives values of $\varepsilon_*(\dot\varepsilon_0)$ calculated by formula (10) for $\dot\varepsilon_{00} = 0.002$ sec^{-1}, $n = 0.15$, $\beta = 8.47$ sec^n, and $C = 0.723 \cdot 10^{-3}$ sec$^{-1/2}$. In Fig. 1, the dependence $\varepsilon_*(\dot\varepsilon_0)$ is shown by a dashed curve.

Thus, the experimentally observed [6] nonmonotonic dependence of the ultimate fracture strain on the rate $\dot{\varepsilon}_0$ with an internal maximum can be described by introducing the dependence of the rate of damage accumulation $\dot{\omega}$ on the logarithmic strain rate $\dot{\varepsilon}_0$ in the form (9). To estimate the total difference between the experimental ε_* and theoretical $\varepsilon_*(\dot{\varepsilon}_0)$ ultimate strains, we introduce the sum

$$S = \sum_{i=1}^{9} \Big(\frac{\varepsilon_* - \varepsilon_*(\dot{\varepsilon}_0)}{\varepsilon_* + \varepsilon_*(\dot{\varepsilon}_0)} \Big)^2. \tag{11}$$

Calculations show that the use of (10) and (11) gives the value $S = 0.0126$.

Let us consider modeling of the nonmonotonic dependence $\varepsilon_*(\dot{\varepsilon}_0)$ by means of two damage parameters. As is known, the nature of fracture depends on the loading parameter (σ_0 or $\dot{\varepsilon}_0$) and, for some materials, it can be different at the same temperature. For large values of σ_0 or $\dot{\varepsilon}_0$, irreversible shear strains develop to fracture which, occurs mainly along grains. For small values of σ_0 or $\dot{\varepsilon}_0$, pore formation occurs along grain boundaries, and pore aggregates then develop into cracks, resulting in integranular fracture of the material. It is obvious that in the cases where there are structure disturbances of both types, it is reasonable to use two structural parameters: $\omega(t)$ and $\Omega(t)$ [7–9].

Let us consider the simplest form of the kinetic equations describing the variation of the structural parameters ω and Ω in time t:

$$\dot{\omega} = D(\dot{\varepsilon}_0/\dot{\varepsilon}_{00})^n, \qquad \dot{\Omega} = D(\dot{\varepsilon}_0/\dot{\varepsilon}_{00})^k, \qquad n > k > 0 \tag{12}$$

(the parameters ω and Ω vary from zero at the initial time to unity at fracture). The moment of fracture t_* is determined by a certain relation between the parameters ω and Ω. As the simplest fracture condition, we adopt the following:

$$\max\left(\omega(t_*), \Omega(t_*)\right) = 1. \tag{13}$$

Relations (12) show that the structural parameters $\omega(t)$ and $\Omega(t)$ vary in time independently of each other. Fracture occurs at the time t_* when, according to (13), the parameter $\omega(t)$ takes a value equal to unity. Under the assumption that the quantity $\dot{\varepsilon}_{00}$ is known from experiments, the two-parameter model of long-term fracture includes three constants: D, n, and k. Integration of (12) from $t = 0$ to t_* yields the following relations for the fracture times $t_{\omega *}$ and $t_{\Omega *}$ and the fracture strains $\varepsilon_{\omega *}$ and $\varepsilon_{\Omega *}$ calculated from the condition of equality of the damage parameters ω and Ω to unity:

$$\begin{aligned} t_{\omega *} &= (\dot{\varepsilon}_{00}/\dot{\varepsilon}_0)^n/D, \qquad \varepsilon_{\omega *} = (\dot{\varepsilon}_{00})^n(\dot{\varepsilon}_0)^{(1-n)}/D, \\ t_{\Omega *} &= (\dot{\varepsilon}_{00}/\dot{\varepsilon}_0)^k/D, \qquad \varepsilon_{\Omega *} = (\dot{\varepsilon}_{00})^k(\dot{\varepsilon}_0)^{(1-k)}/D. \end{aligned} \tag{14}$$

Because $n > k$, from (14) it follows that

$$\begin{aligned} \varepsilon_* &= \varepsilon_{\Omega *} = (\dot{\varepsilon}_{00})^k(\dot{\varepsilon}_0)^{(1-k)}/D \qquad \text{at} \quad \dot{\varepsilon}_0 \leqslant \dot{\varepsilon}_{00}, \\ \varepsilon_* &= \varepsilon_{\omega *} = (\dot{\varepsilon}_{00})^n(\dot{\varepsilon}_0)^{(1-n)}/D \qquad \text{at} \quad \dot{\varepsilon}_0 \geqslant \dot{\varepsilon}_{00}. \end{aligned} \tag{15}$$

Table 1 gives values of the fracture strain calculated from formula (15) for $\dot{\varepsilon}_{00} = 0.003$ sec^{-1}, $D = 1.32 \cdot 10^{-3}$ sec^{-1}, $n = 1.21$, and $k = 0.90$. The dependence $\varepsilon_*(\dot{\varepsilon}_0)$ determined according to (15) for the specified values of the constants is shown in Fig. 1 by a dot-and-dashed curve. Calculation of the total difference between the experimental and theoretical values of ε_* using (11) and (15) shows that in this case, $S = 0.0083$.

Thus, modeling of the ultimate strain using two damage parameters [the kinetic equations (12)] leads to a smaller total spread in theoretical and experimental values of $\varepsilon_*(\dot{\varepsilon}_0)$ than modeling with one damage parameter using the kinetic equation (9).

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